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ON A TAKAGI-SUGENO FUZZY CONTROLLER WITH NON-HOMOGENOUS DYNAMICS

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Abstract: The paper proposes a Takagi-Sugeno fuzzy controller with non-homogenous controller dynamics with respect to the two input channels, that means, in the linear case, different transfer functions with respect to the reference input and to the controlled output. The considered controller is dedicated to a class of third-order integral-type plants, specific to the field of electrical drives, which can be characterised in their simplified linearised forms by standard models. For these models even conventional linear control structures give satisfaction. There is proposed a development method for the fuzzy controller, based on the fact that fuzzy controllers can be, in some certain conditions, well approximated by linear controllers and, so, the Extended Symmetrical Optimum (ESO) method and the Modified Structure of ESO Method are applicable in this situation. The fuzzy controller and its corresponding development method are validated by an application example that can correspond to the speed control of an electrical drive.

Keywords: phase margin, Extended Symmetrical Optimum method, Modified Structure of Extended Symmetrical Optimum method, Takagi-Sugeno fuzzy controller, non-homogenous dynamics.

1. INTRODUCTION

In the case of conventional linear control structures, the development principle that ensures a maximum phase margin or a minimum guaranteed phase margin for some applications (Preitl and Precup, 1999), based on an extension of the relations specific to the Symmetrical Optimum (SO) method after Kessler (Åström and Hägglund, 1995), cannot give fully satisfaction in all applicable situations. The reason for this is in the presence of a zero in the closed-loop transfer function with respect to the reference input $H_w(s)$, that, even for large values of the design parameter ß (ß>10), results in an approximately s_1 = 10% overshoot, much more smaller in comparison with the classical case, SO (with ß=4, s_1 = 43%).

The considered class of plants in this paper is of third-order with integral character, specific to electrical drives especially if continuous modifications of the angular velocity set-point are required (for example, a constant linear velocity is necessary in the applications of rolling mills with the result in variable moment of inertia (Grimble and Hearns, 1999)).

The use in this case of some control system (CS) structures involving controllers with non-homogenous dynamics with respect to the two input channels (of reference input w, and of controlled output y) can give increased satisfaction. This fact is emphasised if the tuning of controller parameters employs the relations specific to a method named the Extended Symmetrical Optimum (ESO) method (Preitl and Precup, 2000).

For the sake of CS performance enhancement, the fuzzy controllers (FCs, originally nonlinear elements without dynamics) must contain dynamics by employing the knowledge on PD, I, PI or PID controllers. Furthermore, in some well stated conditions the approximately equivalence between

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linear and fuzzy controllers is generally acknowledged and used (Tang and Mulholland, 1987; Galichet and Foulloy, 1995), and the PD-, I-, PI- or PID-type fuzzy controllers can be considered in the design phase as nonlinear but linearisable.

In this context, the paper proposes a Takagi-Sugeno fuzzy controller (TS-FC) with non-homogenous dynamics with respect to the two input channels together with a development method. The method is based on linearisation and on the development results from the conventional linear case. Furthermore, the use of a PI-fuzzy controller on one of the channels does not compensate exactly the large time constant of controlled plant, and this requires an additional block connected in parallel.

The paper has the following structure. The conventional CS structure with controller having non-homogenous dynamics with respect to the two input channels and aspects related to its development are briefly discussed in Section 2. Then, the proposed TS-FC with non-homogenous dynamics with respect to the two input channels is analysed in Section 3 resulting also in a development method. There are illustrated in Section 4 the TS-FC and its development method by a case study that can correspond to the speed control of an electrical drive, and the final part of the paper is focussed on the conclusions.

2. CONTROL SYSTEM STRUCTURE AND DEVELOPMENT IN THE LINEAR CASE

The analysed control system structure containing controllers with non-homogenous dynamics with respect to the two input channels is presented by Preitl and Precup (2000) in Fig.1, where: C – controller characterised by the transfer functions $H_{Cw}(s)$ (with respect to the reference input w) and $H_{Cy}(s)$ (with respect to the controlled output y), Fw – correction (reference) filter, CU – comparing unit, $w^{\sim}$ – filtered reference input, e – control error, u – control signal, v, v_1, v_2 – load disturbance inputs (v_1 and v_2 – additive disturbances fed to the plant input and output, respectively), RCM – reference correction module having as inputs the strictly speaking reference input (the set-point) w_0 and the correction reference input $w_{y,v}$ depending on either the controlled output or the load disturbance inputs.

The dynamics of the blocks that make up the controller depend on the plant transfer function, $H_P(s)$. For the accepted case the plant is a third-order system with integral character with the following transfer function, $H_P(s)$:

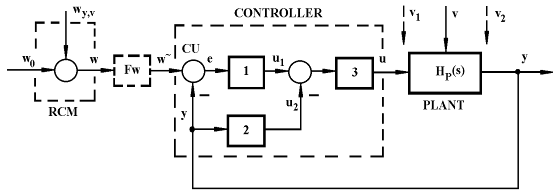

Fig.1. Structure of conventional control system.

$$H_P(s) = \frac{k_P}{s(1+sT_1)(1+sT_\Sigma)}, \quad (1)$$

where k_P is the plant gain, T_S ($T_S < T_1$) stands for the sum of all parasitic time constants of controlled plant, and T_1 represents the large time constant of controlled plant.

The blocks 1 … 3 from Fig.1 are characterised by the transfer functions $H_1(s)$ … $H_3(s)$, respectively:

$$H_1(s) = 1 + \frac{1}{sT_i}, \ H_2(s) = sT_d, \ H_3(s) = K_C. \quad (2)$$

Accordingly, the transfer function with respect to the reference input, $H_{Cw}(s)$, and the transfer function with respect to the controlled output, $H_{Cy}(s)$, can be computed as:

$$H_{Cw}(s)=\frac{k_c}{s}(1+sT_i), \ H_{Cy}(s)=\frac{k_c}{s}(1+sT_c)(1+sT_c'), \quad (3)$$

with the connections between the coefficients of the transfer functions from (2) and (3), for $T_i = 4T_d$, expressed as in (4):

$$k_c=K_C/T_i, \ T_c, T_c'=0.5[T_i\pm(T_i^2-4T_iT_d)^{1/2}], \ T_c<T_c'. \quad (4)$$

By applying for the given plant the principle for compensation of large time constants:

$$T_c' = T_1, \quad (5)$$

the closed-loop transfer function with respect to the reference input, $H_w(s)$, can be expressed as (6) in the absence of Fw:

$$H_w(s) = \frac{1}{1+sT_c+s^2/k_0+s^3T_S/k_0} = \frac{1}{a_0+a_1s+a_2s^2+a_3s^3}, \quad (6)$$

with the open-loop gain k_0 of the CS:

$$k_0 = k_ck_P. \quad (7)$$

By imposing to the coefficients of the transfer function (4) the general conditions specific to the ESO method imposed by Preitl and Precup (1999):

$$ß^{1/2}a_0a_2 = a_1^2, \ ß^{1/2}a_1a_3 = a_2^2, \quad (8)$$

the following relations will be obtained:

$$k_0 = 1/(ß \bullet ß^{1/2} T_S^2)\ ,\ \ T_c = ßT_S\ . \qquad (9)$$

Consequently, the closed-loop transfer function with respect to the reference input

$$H_w(s)_{opt} = \frac{1}{1+ßT_S s + ß\bullet ß^{1/2} T_S^2 s^2 + ß\bullet ß^{1/2} T_S^3 s^3}\ , \qquad (10)$$

where $ß > 1$ is the design parameter. For $ß = 4$ the Kessler's SO method is found as particular case (Åström and Hägglund, 1995).

The following significant aspects can be highlighted for this control system structure. The tuning relations for controller parameters are (11):

$$k_c = 1/(ß\bullet ß^{1/2} T_S^2 k_P)\ ,\ \ T_c = ßT_S\ ,\ \ T_c' = T_1\ . \qquad (11)$$

The phase margin f_r and the crossover frequency $?_c$ can be computed by means of the following relations:

$$f_r = p/2 - \mathrm{arctg}[(ßx)^{1/2}/(1-ß^{1/2}x)]\ ,\ \ ?_c = x^{1/2}/T_S\ , \qquad (12)$$

with x being the real solution belonging to the interval $(1/ß^{5/2},\ 1/ß^{3/2})$ of the third-order equation (13):

$$ß^3x^3 + ß^2(ß - 2ß^{1/2})x^2 + ß^2x - 1 = 0\ . \qquad (13)$$

The controller structure ensures the suppression of the zero from the closed-loop transfer function (10). This is an advantage that can be obtained in the case of homogenous dynamics (meaning the same transfer functions with respect to w and y) (Åström and Hägglund, 1995) only by using adequate reference filters, Fw. Two versions of such filters are proposed by (Preitl and Precup (1999), the simplest one having the transfer function (14):

$$H_{Fw}(s) = 1/(1+ßT_S s)\ . \qquad (14)$$

The design parameter ß is chosen by the designer during the development phase in a recommended domain (Fig.2, $1 < ß = 20$) in accordance with different goals to be reached.

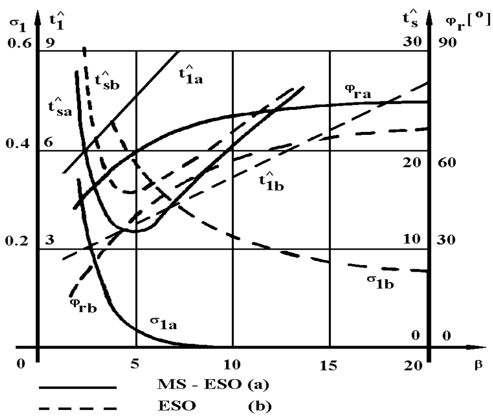

Fig.2. Control system performance indices versus ß.

The control system performance indices (s_1 – overshoot, f_r – phase margin, and in normalised form, $t_1^{\wedge}=t_1/T_S$ – first settling time, and $t_s^{\wedge}=t_s/T_S$ – settling time, and others as well) can be illustrated by the connections as function of ß from Fig.2, where MS-ESO (Modified Structure of Extended Symmetrical Optimum, the case a of Fig.2) means the application of the ESO method to the modified CS structure with non-homogenous dynamics of the controller.

With respect to the classical structure with controllers tuned by the ESO method, the tuning by the MS-ESO method proves to have the advantages illustrated in Fig.2. On the other hand, by comparing the ESO and MS-ESO methods with other analytical tuning methods (see, for example, those (Loron, Voda and Landau, 1995; Loron, 1997)), it can be seen that the ESO and MS-ESO offer at least more complete results.

In the case of using fuzzy controllers according to the CS structure presented in Fig.1, the compensation problem (5) disappears because the controller becomes nonlinear and T_c' is variable depending on the current operating point. This is the reason why there is proposed here a fuzzy controller comprising a PI-fuzzy controller in parallel with a lead-lag element for the compensation of T_1, to be presented in the following Section. There was chosen a TS-FC (Takagi and Sugeno, 1985) due to its property of blending several linear controllers depending on the regions of the input space resulting in bumpless transfer from one linear controller to another.

3. FUZZY CONTROLLER STRUCTURE AND DEVELOPMENT

For obtaining the structure of the TS-FC it is firstly necessary to re-organise the controller structure from Fig.1. By taking into account the diagram from Fig.1 and the relations (2) and (3), the Laplace transform of control signal, u(s), can be expressed as function of the Laplace transforms of control error, e(s), and controlled output, y(s), in terms of (15):

$$u(s) = H_C(s)e(s) - k_cT_isT_dy(s)\ , \qquad (15)$$

with $H_C(s)$ being the transfer function of the linear controller with respect to the control error and having essential effect on CS behaviour:

$$H_C(s) = k_cT_i\left(1+\frac{1}{sT_i}\right)\ , \qquad (16)$$

where the expression of T_i results from (4) and (11):

$$T_i = T_c + T_c' = T_c + T_1\ . \qquad (17)$$

The relation (15) points out a linear PI controller having the transfer function (16). This controller can be replaced, for the sake of CS performance enhancement, by a PI-fuzzy controller. But, in this conditions, the compensation of T_1 will no more be fulfilled. For avoiding this, there is added a PD term

$(1+sT_{c1})$ to the transfer function (16) resulting in its modified form, $H_C^*(s)$, of PID type:

$$H_C^*(s)=k_cT_i(1+\frac{1}{sT_i}+1+sT_{c1})=\frac{k_c}{s}(1+sT_{c2})(1+sT_{c3}). \quad (18)$$

The connections between the coefficients from (18) are:

$$T_{c2} + T_{c3} = 2T_i\ ,\ \ T_{c2}T_{c3} = T_iT_c\ . \quad (19)$$

By imposing for compensation:

$$T_{c2} = T_1\ , \quad (20)$$

and by using (17) and (19), the expression of the time constant of the PD term, T_{c1}, will be:

$$T_{c1} = T_1(T_1+2T_c)/(T_1+T_c)\ . \quad (21)$$

The TS-FC structure results by using the fact that from (16) and (18) it results that:

$$H_C^*(s) = H_C(s) + k_cT_i(1+sT_{c1})\ . \quad (22)$$

The PI term from (20) will be replaced by a PI-FC and, for a further implementation, the PD term from (20) will be replaced by a lead-lag element. Then, by using the dependence (15) and by replacing the derivative element by a real derivative element, the TS-FC structure will be that from Fig.3. The fuzzy control system structure is presented in Fig.1, where the controller is replaced by the TS-FC.

According to Fig. 3, the dynamics is introduced in the structure of the fuzzy controller (FC is the strictly speaking fuzzy controller, without dynamics) by differentiating the control error (e_k) and integrating the increment of control signal ($? u_{1k}=u_{1k}–u_{1k-1}$). Fig.3 highlights the non-homogenous information processing in the controller together with the hybrid information processing (part of the linear parts of the controller are considered as continuous-time systems).

The considered TS-FC is a type III fuzzy system according to (Koczy, 1996; Sugeno, 1999), and in the development phase the input membership functions are initially of regularly distributed triangular type with an overlap of 1 (e_k and $? e_k=e_k–e_{k-1}$ – the increment of control error are the two inputs).

The FC uses the MAX and MIN operators in the inference engine, assisted by the rule base expressed by the decision table from Table 1, and employs the weighted average method for defuzzification (Babuska and Verbruggen, 1996).

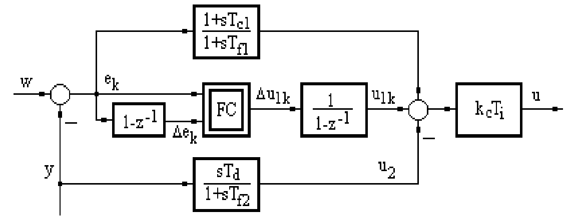

Fig. 3. Structure of TS-FC.

Fig. 4 and Table 1 the strictly positive parameters of the TS-FC to be determined in the sequel by means of the proposed development method: {k_c, T_i, T_d, T_{f1} and T_{f2}} for the linear part and {B_e, $B_{?e}$, ?} for the FC block. The role of the parameter ? is to introduce additional nonlinearities that can be useful for CS performance enhancement especially when controlling complex plants, where the plant (1) can be seen as a sub-system.

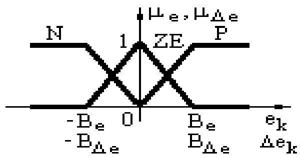

Fig. 4. Accepted membership functions of input linguistic variables.

Table 1 Decision table of FC

		e_k		
		N	ZE	P
	P	$? u_{1k}$	$? u_{1k}$	$?·? u_{1k}$
$? e_k$	ZE	$? u_{1k}$	$? u_{1k}$	$? u_{1k}$
	N	$?·? u_{1k}$	$? u_{1k}$	$? u_{1k}$

For the development of M-PI-FC and TS-PI-FC it is firstly necessary to discretise the continuous linear PI controller with the transfer function (23):

$$H_{PI}(s) = 1+\frac{1}{sT_i}\ . \quad (23)$$

The use of Tustin’s discretisation method leads to the incremental version of the quasi-continuous digital PI controller:

$$? u_{1k}=K_P? e_k + K_Ie_k\ ,\ \ K_P=1–h/(2T_i)\ ,\ \ K_I=h/T_i\ , \quad (24)$$

where h represents the sampling period, and k stands for the current sampling interval.

For ensuring the quasi-PI behaviour of the FC block in the conditions of the accepted rule bases, there is applied the modal equivalences principle (Galichet and Foulloy, 1995), resulting in:

$$B_{?e} = (K_I/K_P)B_e\ . \quad (25)$$

Therefore, there are two degrees of freedom in the development of the FC block, represented by the strictly positive parameters B_e and ?.

By using all aspects presented before, the proposed development method for the Takagi-Sugeno fuzzy controller consists of the following development steps to be proceeded:
a) express the simplified mathematical model of controlled plant in the form (1);
- proceed the phase I corresponding to the linear part of the TS-FC and comprising the steps b) … f):
b) choose the value of the design parameter ß as a compromise between the desired / imposed control system performance by using the diagrams (for s_1, t_1, t_s and f_r) illustrated in Fig. 2;
c) obtain the tuning parameters $\{k_c, T_c, T_c'\}$ of the linear controller with non-homogenous information processing with respect to the two input channels by applying the design relation (11);
d) compute T_i by using (17) and T_d by using the following relation obtained from (4) and (17):

$$T_d = T_c T_c'/T_i = T_c T_1/(T_c + T_1) \,. \qquad (26)$$

e) apply (21) for computing the time constant T_{c1};
f) choose the sufficiently small filtering time constants T_{f1} and T_{f2} in relation with the other time constants involved in the CS (Ogata, 1990);
- proceed the phase II corresponding to the nonlinear part of the TS-FC (the FC block) and comprising the steps g) and h):
g) choose a sufficiently small sampling period, h, accepted by quasi-continuous digital control, take into account the presence of a zero-order hold (Isermann, 1977), discretise the continuous-time linear PI controller (23) and compute the parameters $\{K_P, K_I\}$ of the resulted quasi-continuous digital PI controller from (24);
h) choose the values of the parameters B_e and ? of the FC block, in accordance with the experience of an expert in control systems or by means of adaptive structures (Wang, 1994), and use (25) to obtain the value of $B_{?e}$.

4. CASE STUDY

For the validation of the proposed fuzzy controller and of development method, there is considered a case study that correspond to the speed control of a separately excited DC drive, where the controlled plant is characterised in its linearised version by the transfer function (1), in normalised form, with the parameters k_P=1, T_S=1 sec and T_1=5 sec.

The development steps presented in the previous Section result in the values of parameters briefly presented as follows.

Firstly, the linear part of the Takagi-Sugeno fuzzy controller is developed by proceeding the phase I of the development method. Therefore, there is chosen the design parameter ß = 16 (that ensures good CS performance in the linear case), and from (11) there will result: k_c = 0.0156, T_c = 16 sec, T_c' = 5 sec. Then, from (17), (26) and (21) there will be obtained T_i = 21 sec, T_d = 3.8095 sec and T_{c1} = 8.8095 sec, respectively. Finally, there are chosen the filtering time constants, T_{f1} = 0.3 sec and T_{f2} = 0.3 sec.

Then, there is developed during the phase II the FC block as part of the TS-FC. For the sampling period h = 0.02 sec, the parameters from (24) take the values K_P = 0.9995 and K_I = 0.00095. By choosing (for a unit step modification of the reference input) B_e = 0.3, the relation (25) results in $B_{?e}$ = 0.00028. By using the recommendation proposed by Precup and Preitl (1998) for a Mamdani fuzzy controller:

$$? \in (0, 1] \,, \qquad (27)$$

there is chosen ? = 0.95.

Part of the digital simulation results is presented in Fig.5 and Fig.6 for the CS with non-homogenous linear PID controller (the structure from Fig.1 involving the transfer functions (2)), and in Fig.7 and Fig.8 for the CS with the developed Takagi-Sugeno fuzzy controller (TS-FC). There was taken into consideration the following simulation scenario: a unit step modification of reference input w, followed by a –0.5 step modification of disturbance input v_2 (after 75 sec), with continuous line for y, and dotted line for u.

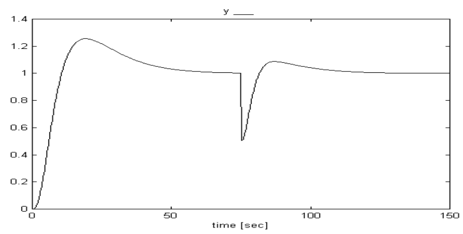

Fig. 5. Controlled output versus time for CS with non-homogenous linear PID controller.

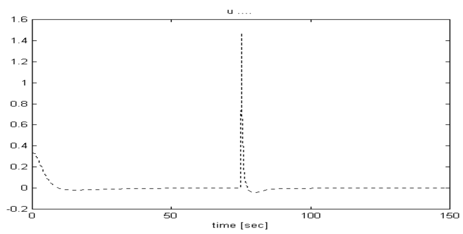

Fig. 6. Control signal versus time for CS with non-homogenous linear PID controller.

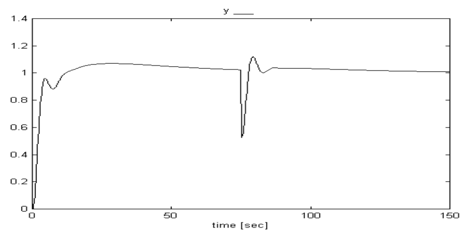

Fig. 7. Controlled output versus time for CS with Takagi-Sugeno fuzzy controller.

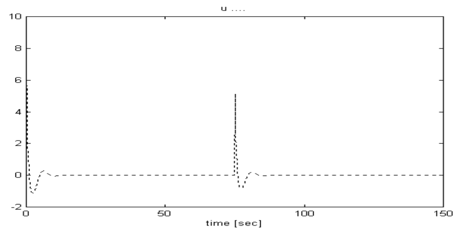

Fig .8. Control signal versus time for CS with Takagi-Sugeno fuzzy controller.

5. CONCLUSIONS

The paper proposes a Takagi-Sugeno fuzzy controller with non-homogenous dynamics with respect to the two input channels together with its development method meant for controlling a class of third-order systems with integral character.

The development method comprises useful development steps, and it is mainly based on applying the tuning relations specific to the MS-ESO method to the original non-homogenous linear PID controller. Then, there is applied the modal equivalences principle, and some linear dynamics is added to its structure for better compensation of the large time constant of controlled plant.

There was developed a TS-FC for a case study, and the presented digital simulation results validate the proposed fuzzy controller and development method. The control system performance enhancement ensured by the proposed fuzzy controller proves that the TS-FC can successfully cope with control of plants from the field of electrical drives. Control system performance can be further improved by using reference filters Fw.

The proposed fuzzy controller is very simple and the development method is transparent and flexible. Therefore, the TS-FC can be further used as low cost automation solution in control of complex plants when several situation occur corresponding to plants which can be approximated by the considered plant (1), or can contain the plant (1) on the lower hierarchical level.

From this point of view, fuzzy control is not seen in this paper as a goal in itself, but as a remarkable low cost automation solution ensuring control system performance enhancement.

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